Einstein's 1935 papers: EPR=ER?

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Abstract

In May of 1935 Einstein published with two co-authors the famous EPR-paper about entangled particles, which questioned the completeness of Quantum Mechanics by means of a gedankenexperiment. Only one month later he published a work that seems unconnected to the EPR-paper at first, the so called Einstein-Rosen-paper that presented a solution of the field equations for particles in the framework of general relativity. Both papers ask for the conception of completeness in a theory and from a modern perspective it is easy to believe that there is a connection between these topics. We question whether Einstein might have considered that a correlation between nonlocal features of Quantum Mechanics and the Einstein-Rosen bridge can be used to explain entanglement. We analyse this question by discussing the used conceptions of "completeness", "atomistic structure of matter", and "quantum phenomena". We discuss the historical embedding of the two works and the context to modern research. Recent approaches are presented that formulate a EPR=ER principle and claim an equivalence of the basic principles of these two papers.

1 Introduction

The historical approach to the foundations of physics is often misunderstood as an historical work alone. To study specific questions in the history of physics is a method of introspection about problems that are still relevant to recent physics. This work is not intended to be a historical work alone. Recent aspects of different conceptions of completeness will be discussed starting from an historical perspective. The connection between these two papers that at first look seem to be independent is presented and gives important insights on Einstein's conception of reality and the recent advent of the EPR=ER concept.

The year 1935 is a remarkable year in the context of Quantum Mechanics. The discussion on the incompleteness of Quantum Mechanics found its culmination in the famous EPR paper [Einstein et al., 1935] and the reply by [Bohr, 1935]. Further [Schrödinger, 1935] expressed his criticism on the oddities of Quantum Mechanics with his famous Gedankenexperiment of a cat in a box.

At first view unconnected to the discussion in Quantum Mechanics, [Einstein and Rosen, 1935b] published one month later an article which is famous as well, but in a completely different field of physics. It is the paper that presented the Einstein-Rosen bridge solution in General Relativity, which later became well known as "wormhole" in popular science and science fiction.

We will show that the EPR paper cannot be seen stand-alone, but must be seen in context with the [Einstein, 1934] paper and the [Einstein and Rosen, 1935b] paper.

In 1934 Einstein expressed his point of view on physical theories in general, the formation of new theories, the connection to an ontology, and to the concept of reality. Later articles and letters from Einstein will help to complete the picture about his views on the nature of reality, and his notion of completeness.

It is tempting to think that the problem Einstein illustrated with the EPR paper is meant to be solved with the Einstein-Rosen bridge solution by solving the occurring nonlocality[1] in Quantum Mechanics with a two-sheet wormhole that represents the entangled particles. After analysing the EPR paper and the notions of reality and completeness the idea of a connection between those papers will be discussed in context with recent developments in the field.

2 Einstein's view on theories and theory formation

In 1934 Einstein was driven by the long lasting debates on the foundations of Quantum Mechanics to clarify his views on the method of theoretical physics in general. He emphasized the nature of the constituents of a theory used in physics [Einstein, 1934]:

> "For to the discoverer in that field, the constructions of his imagination appear so necessary and so natural that he is apt to treat them not as the creations of his thoughts but as given realities."

and

> "He is in just the same plight as the historian, who also, even though unconsciously, disposes events of the past around ideals that he has formed about human society."

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[1]It is still a matter of disputes in the interpretations of Quantum Mechanics if there is nonlocality in Quantum Mechanics. We define a nonlocal theory as a theory that violates the Bell inequalities

This already indicates that the objection that Einstein was a proponent of a naive realism[2] has to be reconsidered. His ontology is of course not at trivial one and we have to define what is meant exactly by naive realism. We will come back later to that discussion.

Einstein then mentions the second elementary truth science is based upon. Its dependency on experience, the need for empirical verification:

> "Pure logical thinking can give us no knowledge whatsoever of the world of experience; all knowledge about reality begins with experience and terminates in it."

It is remarkable that Einstein proposed his views parallel to [Popper, 2005] and they agree in several points. We will see later that their views are connected more deeply. What are those ideals that Einstein is referring to? In his view a theory is built upon concepts and laws and has to obey certain requirements:

> "The basic concepts and laws which are not logically further reducible constitute the indispensable and not rationally deducible part of the theory. It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience"

Einstein is referencing to Ockham's razor principle in this statement on the economy, the theory building process has to comply with. A detailed analysis of Ockham's razor in context to Quantum Mechanics is given by [Krizek, 2017b].

Einstein's ideas on theory building are guided by the structure of the general Theory of Relativity and its success. To Einstein it was representative to show the incorrectness of the induction principle as a theory building process and to the contrary consolidated his beliefs in a more top down orientated theory building process out of principles.

By rejecting induction as theory building principle and taking up the position that the elements of theories are creations of human thoughts, the question arises if there is a correct way to find insights about nature. And here Einstein makes clear that to him there is such a way [Einstein, 1934]:

> " To this I answer with complete assurance, that in my opinion there is the correct path and, moreover, that it is in our power to find it. Our experience up to date justifies us in feeling sure that in Nature is actualized the ideal of mathematical simplicity. It is my conviction that pure mathematical construction enables us to discover the concepts and the laws connecting them which give us the key to the understanding of the phenomena of Nature."

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[2]See[Janssen and Lehner, 2014, page 309ff]

His argument in favour of this position is the huge success of mathematics as a tool to describe our experiences about nature. It is this unreasonable effectiveness of mathematics that lead Eugene Wigner to publish his paper [Wigner, 1960] of the same title. On the other hand Einstein identified a problem in the above argument, when he states that this mentioned success is claimed as well by theories that cannot deliver a deeper ontological insight such as classical mechanics.

> "Have we any right to hope that experience will guide us aright, when there are theories (like classical mechanics) which agree with experience to a very great extent, even without comprehending the subject in its depths?"

Einstein ignores this objection and heads for a worldview with a distinguished position of mathematics:[3]

> "...the truly creative principle resides in mathematics. In a certain sense, therefore, I hold it to be true that pure thought is competent to comprehend the real, as the ancients dreamed."

It reflects a kind of mathematical realism which is in one aspect contradictory, when Einstein pointed out that the elements of the theory are ideals, are pure constructions of human thoughts, but on the other hand represent a "correct way", reflecting a truth about nature.
As basic entity of this world-view he adopts the four dimensional continuum, which already was extremely successful for unification of physics during the end of the 19th century. He refers to this success and the principle of simplicity and states:

> "It is essential for our point of view that we can arrive at these constructions and the laws relating them one with another by adhering to the principle of searching for the mathematically simplest concepts and their connections. In the paucity of the mathematically existent simple field-types and of the relations between them, lies the justification for the theorist's hope that he may comprehend reality in its depths."

Einstein presented and clarified his ideas on theory formation in general, about his ontological ideas and the scientific method in general. He presented field theories as candidates for this simple theory that comprehends the real. His concern was mostly about how to integrate the atomistic structure[4] of matter and energy.

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[3]Wigner will recall to Einstein's objection 25 years later.
[4]In todays notion we would call it quantum nature

We will see later that he used this wording again in his 1935 papers and it is relevant to see what he meant exactly:

> "...the theory[5] in its basic principles is not an atomic one in so far as it operates exclusively with continuous functions of space, in contrast to classical mechanics whose most important feature, the material point, squares with the atomistic structure of matter."

What Einstein understands as "atomic theory" or "atomistic structure" is discrete localization in space in contrast to continuous functions of fields in space. And he emphasizes the importance of this idea in the conclusion of the paper:

> "Thus in a continuum theory, the atomistic character could be satisfactorily expressed by integral propositions without localizing the particles which constitute the atomistic system. Only if this sort of representation of the atomistic structure be obtained could I regard the quantum problem within the framework of a continuum theory as solved."

To Einstein this concept is of great importance to represent particles in a field theory. The singularity-free localization of particles is a core element of a successful theory. Some aspects of Einstein's views on the structure of theoretical physics directly relate to the nature of reality. We will now analyse the notions of Einstein's reality and the structure of theories.

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[5]He refers to the field theory

3 Einstein's not-naive realism and the notions of reality

3.1 A classification scheme for physical theories

A classification scheme is presented to categorize different aspects of a theory. Therefore we define graphical representations of physical quantities, physical laws, physical concepts and elements of reality.

.	Physical quantity
•——•	Law between physical quantities
(icon)	Concept connecting laws and physical quantities
◇	Elements of objective reality

Table 1: Icons used in the classification scheme

Out of these icons we can construct a depiction for a theory and for elements of reality inside objective reality.
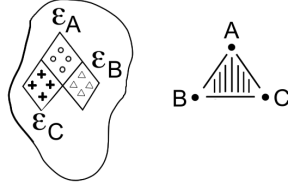
Figure 1: Elements of reality and the structure of a theory

Formally we define a set of elements of a theory T_{PR} and a set of elements of reality PR.

$$T_{PR} = \{t_i\} = \{A, B, C\} \tag{1}$$

$$PR = \{\epsilon_i\} = \{\epsilon_A, \epsilon_B, \epsilon_C\} \tag{2}$$

We will make use of them in the analysis of the reality criterion and the conceptions of completeness.

For classification we propose four levels in a theory to classify physical theories regarding their different aspects. The intention is to compare different interpretations of Quantum Mechanics regarding their formalism, concepts and ontologies.

- The first level contains the mathematical formalism of the theory, mathematical symbols and the connecting laws. The mathematical symbols can be identified with symbols for physical quantities, so we will speak of physical quantities though we mean only their symbolic representations.

- The second level contains the assignment of the before defined physical quantities to sensations. Physical quantities consist of quantities and units and need a procedure to be measured. All these assignments are contained in the second level.

- The third level contains concepts and principles that give reason and structure to the theory. First principles, like extremal principles or conservation principles belong to this level.

- The fourth level is the ontological embedding of the theory, it defines how physical quantities and concepts are related to ontological elements. Even if there are no ontological elements like in a positivistic philosophy of science this would be defined here. Here the elements of reality Einstein refers to are located. They are parts of the objective reality.

We will use the classification scheme for the discussion of the concept of completeness, the criterion of reality, and in the analysis of naive realism in the chapter 3.2. A detailed proposal of the classification scheme can found in Krizek [2017a].

3.2   Naive realism

Naive realism, or direct realism is the view that the perception delivers a full mapping between the elements of reality and the sensations. There are a lot of subtleties that could be discussed in that regard, some views of naive realism will require the spatiotemporal reference frame, some views will assume materialistic ideas. We will confine to those aspects that are of interest for our discussion.

We already mentioned the objection that Einstein was a proponent of naive realism. The term "naive" is depreciative inherently and includes prejudices, though will we make use of it due to the historical account, but we distance ourselves from a depreciative intent of this term. To get a better idea about the concept of naive realism we present the definition by [Russell, 1940]:

> "We all start from naive realism, i.e., the doctrine that things are what they seem. We think that grass is green, that stones are hard, and that snow is cold. But physics assures us that the greenness of grass, the hardness of stones, and the coldness of snow are not the greenness, hardness, and coldness that we know in our own experience, but something very different. The observer, when he seems to himself to be observing a stone, is really, if physics is to be believed, observing the effects of the stone upon himself."

Another statement of naive realism by [Einstein, 1946] shows Einstein's position on this question:

> "...naive realism,... according to which things are as they are perceived by us through the senses. This illusion dominates the daily life of men and of animals; it is also the point of departure in all of the sciences, especially of the natural sciences. ... The overcoming of naive realism has been relatively simple."
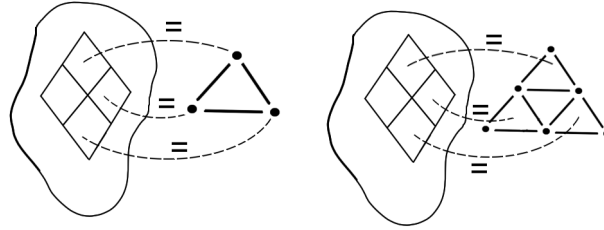

Figure 2: Naive realism and an ontology for quantities and concepts

This last remark in [Einstein, 1946] shows that Einstein would not consider himself as a proponent of naive realism.

If we apply the classification scheme presented before on a definition of naive realism, one would conclude that there are identities between the physical quantities, or the physical concepts with the ontological elements of reality.

In the context of materialism this would mean that the rigid bodies we experience through sensations and we model as a concept within Newtonian mechanics, would be assumed to exist as experienced. This view is rejected by physics and philosophy of science. A detailed discussion on the rejection of direct and naive realism can be found in [Le Morvan, 2004].

But still a kind of naive realism is discussed in recent physics, the mathematical universe hypothesis(MUH) [Tegmark, 2004]:

> "Let us now digest the idea that physical world (specifically, the Level III multiverse) is a mathematical structure. Although traditionally taken for granted by many theoretical physicists, this is a deep and far-reaching notion. It means that mathematical equations describe not merely some limited aspects of the physical world, but all aspects of it. It means that there is some mathematical structure that is what mathematicians call isomorphic (and hence equivalent) to our physical world, with each physical entity having a unique counterpart in the mathematical structure and vice versa."

It is not our goal to discuss the MUH here, but we have seen before that Einstein had in mind a kind of mathematical realism. We will discuss the notions of Einstein's realism later after clarifying some subtleties in the EPR paper.

3.3 Subtleties in the EPR paper

When we discuss reality we have to clarify the used terms and there we immediatly face one subtlety. In the handwritten note for the talk he held in Oxford at the Herbert Spencer lecture on the method of theoretical physics [Einstein, 1933] made use of the two terms "Wirklichkeit" and "Realität" synonymical. In the english translation and published paper [Einstein, 1934] both terms merge to the term "reality". It is clear from the context in which Einstein made use of these terms, that "Wirklichkeit" stands for the objectively given facts in nature, independent of observation. "Wirklichkeit" refers to the ontology in the world. He makes use of "Realität" only once in the context of an allegory to explain his personal struggle with philosophy of science [Einstein, 1934]:

> " For to the discoverer in that field[6], the constructions of his imagination appear so necessary and so natural that he is apt to treat them not as the creations of his thoughts but as given realities."

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[6]Theoretical physics

In [Einstein et al., 1935] we find three wordings, but the paper has been written originally in English: "objective reality", "physical reality" and "reality". The latter is just used as a short form for "physical reality". The definition of "objective reality" can be found in the introduction of the paper:

> "Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates."

It is noticeable that Einstein defines the term "objective reality" and then in the definition of completeness makes use of "physical reality" (see citation in Section 4.3) which has not been defined before [Einstein et al., 1935]:

> "Whatever the meaning assigned to the term complete, the following requirement for a complete theory seems to be a necessary one: every element of the physical reality must have a counterpart in the physical theory"

Out of how the reality criterion refers to this "physical reality" and the elements of reality, and the lack of further definition of "physical reality" we conclude that "physical reality" is synonymous to "objective reality".

Out of Einstein's dissatisfaction with the precipitate submission of the manuscript by Boris Podolsky [von Meyenn, 2010, letter 206, p.537] we conjecture that some flaws remained in the paper.

One more of these flaws is the sloppy use of the assignment of physical quantities to elements of physical reality which we demonstrate by means of three statements from the paper.

Statement 1

> "... there is an element of physical reality corresponding to the physical quantity A."

Statement 2

> "The usual conclusion from this in Quantum Mechanics is that when the momentum of a particle is known, its coordinate has no physical reality."

Statement 3

> "In accordance with our criterion of reality, in the first case we must consider the quantity P as being an element of reality, in the second case the quantity Q is an element of reality."

There is a big difference for the nature of reality in the different statements above. If we apply the classification scheme for the structure of theories given by [Krizek, 2017a] we see that the latter statements would represent a naive realism where the referred physical quantity would be identical with the ontology of the theory.

The first statement that uses the correspondence between the physical quantity and the element of reality is more subtle. There the physical quantity is a mathematical object with interpretation and corresponds to an independent ontological element of reality (see Table 3). The element of reality itself is only comprehensible through sensations, though it is expected to exist in the physical reality but never accessible to us in its whole nature.

It is coherent with Einstein's views on philosophy of science we presented above and with the context of the EPR-paper that he did not favour the point of view presented in the statements 2 and 3.

3.4   Notions of reality

Elements of reality

In the EPR argument and the reality criterion specifically a term is used to describe and speak about properties of objective reality, the term "element of reality". It is interesting to trace back the term "element" to a letter from Einstein to the philosopher Moritz Schlick in which he refers to Ernst Mach [Hentschel, 1986]:

> "Es scheint mir nun, daß das Wort 'wirklich' in verschiedenem Sinne genommen wird, je nachdem es von Empfindungen oder von Ereignissen bzw. Thatbeständen in physikalischem Sinne ausgesprochen wird. Wenn zwei verschiedene Völker unabhängig voneinander Physik treiben, werden sie Systeme schaffen, die bezüglich der Empfindungen ('Elemente' im Sinne Machs) gewiss übereinstimmen. Die gedankliche Konstruktion, die die beiden zur Verknfüpfung dieser 'Elemente' ersonnen, können weitgehend verschieden sein."

> "It seems to me that the term "real"[7] is used in different meaning, depending on whether used in physical context with sensations or on the other hand events respectively facts. When two nations pursue physics independently from each other, they will create systems that agree concerning the sensations(elements in the meaning of Mach). The construction of ideas both of them developed to connect these elements can differ widely.[8]"

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[7] Refer to the discussion before on the terms "wirklich" und "real".

[8] Translation by the author

Is this identification of the Machian elements mentioned by Einstein in the letter to Morith Schlick and the elements used in the EPR argument justified? In the Machian sense an element is defined as sensation, as a representation for events [Mach, 1922]:

> "Die Ansicht, welche sich allmählich Bahn bricht, da die Wissenschaft sich auf die übersichtliche Darstellung des Tatsächlichen zu beschränken habe, führt folgerichtig zur Ausscheidung aller müßigen, durch die Erfahrung nicht kontrollierbaren Annahmen, vor allem der metaphysischen (im Kantschen Sinne). Hält man diesen Gesichtspunkt in dem weitesten, das Physische und Psychische umfassenden Gebiete fest, so ergibt sich als erster und nächster Schritt die Auffassung der Empfindungen" als gemeinsame Elemente" aller möglichen physischen und psychischen Erlebnisse, die lediglich in der verschiedenen Art der Verbindung dieser Elemente, in deren Abhängigkeit voneinander bestehen."

> " The view that wins recognition step by step, that science has to confine to clearly arranged representations of facts, leads to the exclusion of all dispensable, through experience not-controllable assumptions, specifically the metaphysical ones(in the meaning of Kant). Applying this view to physical and psychical fields, as first and next consequence results the view of sensations as common "elements" of all possible physical and psychical experiences. These sensations only exist in their different ways of connection between them and their mutual dependence of each other.[9] "

and further

> "Berkeley[10] sieht die Elemente" als durch etwas außer denselben Liegendes, Unbekanntes (Gott) bedingt an, wofür Kant, um als nüchterner Realist zu erscheinen, das Ding an sich" erfindet, während die hier vertretene Anschauung mit einer Abhängigkeit der Elemente" von einander praktisch und theoretisch das Auskommen zu finden glaubt."

> " Berkley sees the "elements" as caused by something unknown(god) beyond them, Kant on the other hand, to appear as a down-to-earth realist, invents "the thing itself", while the here presented approach of mutual depending elements claims to get along for practical and theoretical purposes.[11] "

Machs philosophy of science is a positivistic one, an Empirio-criticism [Avenarius and Avenarius, 1905] and [Avenarius, 1906]. Metaphysical parts of the theory are rejected as non-physically and unnecessary, also with regard to a principle

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[9]Translation by the author

[10]Mach means the philosopher George Berkley

[11]Translation by the author

of economy of thought all theories have to comply with. The elements Einstein is referring to are those parts of the theory that are indisputable because they represent the experiences, which are independent of the type of physical theory, in Einstein's relativistic context the invariants of a theory.

It is irony, that Einstein refers to the originally positivistic assigned term "elements" to defend his realistic point of view, but he mentions also the Machian elements and gives a clear distinction between Machian elements and elements of reality [Einstein et al., 1935]:

> "In a complete theory there is an element corresponding to each element of reality."

The "element" in the theory is an element in the sense of Mach. The corresponding "element of reality" is not part of the theory, it is part of the ontology, part of the objective reality independent of our perception.

It is strange that the term "element" is never again used by Einstein, not in his explanation of the EPR argument in a letter to Schrödinger [von Meyenn, 2010, Letter 206, p.537], nor in the explanation in a letter to Popper[Popper, 2005] nor in the reformulation of the EPR argument in [Einstein, 1948]. This could give rise to the objection that the term "element" was coined by Podolsky who wrote most of the article as pointed out by[Kiefer, 2015, Chapter 2.4] and in [von Meyenn, 2010, Letter 206, p.537].

The content of the EPR argument has been formulated after long discussions[12] of Einstein, Podolsky, and Rosen, so it seems feasible that the cornerstones of the paper were clearly defined before by all authors.

Secondly though Einstein did not use the term in this context again, he mentions the "elements" before in the 1917 letter to Moritz Schlick [Hentschel, 1986]:

> "...a gap which widens progressively with the developing unification of the logical structure, that is with the reduction in the number of the logically independent conceptual elements required for the basis of the whole system."

He further makes use of them in [Einstein, 1934]:

> "It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible..."

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[12][von Meyenn, 2010, Letter 206, p.537]

It seems that Einstein had reservations about the way the EPR argument was constructed, paired with Podolsky's not-agreed submission of the EPR paper and a parallel press statement raised Einstein's anger [Kiefer, 2015]. It is therefore plausible that the elements are indeed a reference by Einstein to the positivistic Machian "elements", amending them with ontological counterparts, the realistic "elements of reality", but Einstein did not wanted to refer to the EPR-paper and its terminology again.

The reality criterion

The reality criterion is defined at the begin of the EPR paper to start the line of argument and identify physical quantities and concepts that have a counterpart in the objective reality. The problem is to find those quantities and concepts that refer to reality and have a corresponding element in reality. For example the concept of centre of mass of a body is not useful to describe some fundamental real property of the rigid body. It is merely a concept in the theory that is helpful to construct a plain theory and has some practical and technical advantages. So it is needed to find a way to identify elements in the theory that refer to an element in reality. Einstein admits that in his letter to Schrödinger [von Meyenn, 2010, letter 206, p.537]:

> "Die eigentliche Schwierigkeit liegt darin, daß die Physik eine Art Metaphysik ist; Physik beschreibt Wirklichkeit. Aber wir wissen nicht, was Wirklichkeit ist; wir kennen sie nur durch die physikalische Beschreibung!"[13]

> "The difficulty is due to that physics is a kind of metaphysics; physics describes "reality". But we do not know what "reality" is; we only know it through the physical description!"

The identified physical quantities that refer to the elements of reality are then chosen to show the incompleteness of Quantum Mechanics [Einstein et al., 1935]:

> "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

We can define the reality criterion more formally by using the definitions from equations 1 and 2.
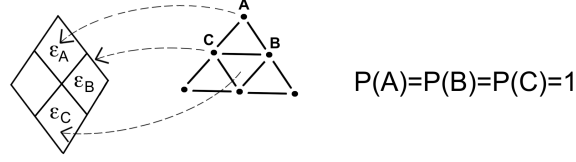
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[13]Translation by the author

Figure 3: Elements of reality

One has do define a probability measure $P(t_i)$ for the elements of the theory. Then the reality criterion can be formally expressed as:

$$\forall t_i \in T_{PR} \wedge P(t_i) = 1 : \exists \epsilon_i \in PR \tag{3}$$

We will have a mapping ϕ that connects the element of the theory with the corresponding element of reality.

$$\phi : T_{PR} \to PR$$

$$\phi(t_A) = \epsilon_A$$

The remarkable thing in the way the reality criterion is defined is the working direction. Out of a probability statement in the theory the reality criterion identifies corresponding elements of reality. If we assume classical physics that fulfils determinism, all physical quantities obey a probability[14] measure of one. Therefore in classical physics all physical quantities should have an corresponding element of reality. The situation is not that trivial as an example will show.

Let us assume Newtonian mechanics in one dimension to describe the motion, i.e. the position of a particle A in reference to a second particle B. The particle is assumed to have a constant mass and is described by a mass point. Our theory then contains the following elements: Occurring forces, the mass of the particle, the position coordinate of particle A and the position coordinate of particle B.

$$T_{PR} = \{F, \ m, \ x_A(t), \ x_B(t)\} \tag{4}$$

The theory will connect these elements with Newton's second law to describe the motion of particle A

$$F = m\frac{d^2 x_A(t)}{dt} \tag{5}$$

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[14]We neglect all interpretational issues concerning probability here and consider probability in classical physics as a propensity.

Let us assume the most simple situation, the force vanishes and becomes zero. This differential equation delivers the equation of motion with two independent constants that refer to the initial conditions. They turn out to be the initial speed of particle A and the initial position of particle A.

$$x_A(t) = v_{Ao} \cdot t + x_{Ao} \qquad (6)$$

Our theory is deterministic so, therefore the probability measure is equal to one, so by applying the criterion of reality we can map all elements of the theory to elements of reality with a mapping ϕ. That way we have identified corresponding elements of reality $PR = \{\epsilon_F, \epsilon_m, \epsilon_{x_A(t)}, \epsilon_{x_B(t)}\}$.
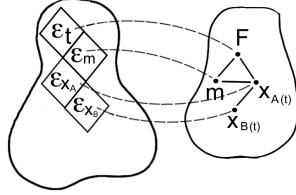

Figure 4: Newton's theory mapped to elements of reality

But there is a problem. Coordinates can be transformed to a different reference frame. Let us apply Galilean transformations on the elements of our theory.

$$x' = x(t) + v \cdot t + a \qquad (7)$$

Without going into details it turns out that force and mass will be invariant under Galilean transformation, but we will receive transformed elements of the theory for $x_A(t)$ and $x_B(t)$. A mapping of the transformed theory T' delivers new elements of reality.
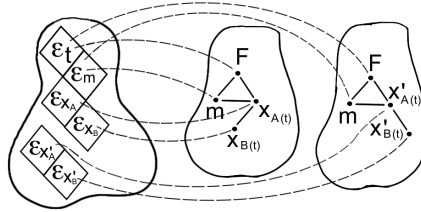

Figure 5: Newton's theory mapped to elements of reality

This mapping shows us that only by using deterministic elements of the theory, and therefore fulfilling Einstein's criterion of reality, we are not able to identify elements of reality that are really satisfying, because they cannot be identified unambiguous.

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What can be done to resolve this unsatisfying situation is to choose the elements of the theory differently. The position of the particle A can also be described by the distance to the particle B.

$$d(t) = x_A(t) - x_B(t) \tag{8}$$

It turns out that this distance $d(t)$ is invariant under Galilean transformation and the mapping ϕ and ϕ' of the theory T and its Galilean-transformed alternative T'. We therefore obtain elements of reality that are satisfying due to their invariance.
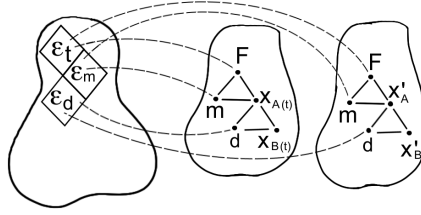

Figure 6: Newtons theory mapped to elements of reality with invariants

The chosen example was simple to demonstrate the idea, but can be transferred to the theory of relativity or any other classical theory. Specially in the theory of relativity the concept of invariance(covariance) will play a foundational role in the conception of the theory. Einstein refers to this invariance principle when he states [Einstein, 1936]:

> "Nur solche Gleichungen sind als Ausdruck allgemeiner Naturgesetze sinnvoll, welche bezüglich in diesem Sinne beliebiger Transformationen kovariant sind. (Postulat der allgemeinen Kovarianz)."

> "Only such equations that fulfil covariance regarding any transformation are resaonable to represent laws of nature. (Postulate of general covariance).[15]"

The reality criterion is a method to find those invariant elements in the theory to identify corresponding elements of reality that have ontological character. Lehner emphasizes the role of invariants in [Lehner, 2014, p.313]:

> "The association of descriptive invariants with objective reality has become a methodological standard in modern physics: it is the conceptual basis not only of general relativity, but of all forms of gauge theories, also in their quantum forms."

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[15]Translation by the author

This key role of invariances will encourage Lehner to define a methodological realism he ascribes to Einstein's philosophy of physics, which we will discuss in section 3.4.

Another aspect of the reality criterion is worth a discussion. [Fine, 2009] states that the reality criterion can only be found in the EPR paper and was not used by Einstein in any other presentation of the EPR argument:

> "...one should note that although Einstein himself later published several versions of EPR, none of them makes any reference to or use of that reality criterion."

We can show that this is not correct. In his letter to Karl Popper Einstein presented the EPR argument in a compact way, but clearly made use of the reality criterion when he states [Popper, 2005]:

> "For if, upon freely choosing to do so [that is, without interfering with it], I am able to predict something, then this something must exist in reality."[16]

The reality criterion is basis to one main agenda in the EPR paper, to criticise the statistical character of Quantum Mechanics.

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[16] The original wording is slightly different and we would prefer to give a different translation:

> "Denn was ich nach freier Wahl prophezeien kann, das muss auch in der Wirklichkeit existieren"

> "What can be predicted freely choosen with conviction, that has to exist in reality."

The statistical character of Quantum Mechanics

In the EPR paper no direct reference can be found to answer the question if in the EPR argument the quantum-mechanical description refers to an statistical ensemble or to single events, to a single quantum-mechanical object. It is not discussed explicitly, though one can conclude for good reason that a single object is meant when they repeatedly speak about a particle's behaviour [Einstein et al., 1935, p.779]:

> *"The fundamental concept of the theory is the concept of state, which is supposed to be completely characterized by the wave function ψ, which is a function of the variables chosen to describe the particle's behaviour."*

> *"For this purpose let us suppose that we have two systems, I and II,... We can then calculate with the help of Schrödinger's equation the state of the combined system I + II at any subsequent time;"*

Using the term "system" the authors left open if they apply the quantum-mechanical description to an statistical ensemble or to a single quantum-mechanical object. One year later, after several discussions and after a reply paper by Bohr, Einstein had chosen a clear position on this question [Einstein, 1936]:

> *"Es scheint mir deshalb klar, dass die Born'sche statistische Deutung der Aussagen der Quantentheorie die einzig mögliche ist: Die ψ-Funktion beschreibt überhaupt nicht einen Zustand, der einem einzelnen System zukommen könnte; sie bezieht sich vielmehr auf so viele Systeme, eine "System-Gesamtheit" im Sinne der statistischen Mechanik."*

> *"It is obvious to me that the statistical interpretation of quantum theory by Born is the only way: The ψ-function does not describe a state that represents one single system; It refers to many systems, to a system-totality in the sense of statistical mechanics."*[17]

Even more he refers to the EPR paper and states that the ensemble interpretation solves any difficulty [Einstein, 1936]:

> *"Eine solche Interpretation beseitigt auch eine yon mir zusammen mit zwei Mitarbeitern jüngst dargestellte Paradoxie ... Die Zuordnung der ψ-Funktion zu einer System-Gesamtheit beseitigt auch hier jede Schwierigkeit."*

> *"Such an interpretation also removes a recently by me and two colleagues presented paradox ... The assignment of the ψ-function to a system-totality here also eliminates all difficulties."*[18]

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[17]Translation by the author
[18]Translation by the author

It is not clearly defined what Einstein had in mind with these "difficulties", but it is most likely that he refers to critiques on the EPR paper [Bohr, 1935]; [Ruark, 1935]. Without any doubt the difficulties do not refer to the question if Quantum Mechanics is complete or incomplete. That position of Einstein is unchanged as one can see from the presented statement concerning the EPR argument one year later [Einstein, 1936]:

> "Diese Zuordnung mehrerer ψ-Funktionen zu demselben physikalischen Zustande des Systems B zeigt wieder, dass die ψ-Funktion nicht als (vollständige) Beschreibung eines physikalischen Zustandes (eines Eingelsystems) gedeutet werden kann."

> "This assignment of several ψ-functions to one physical state of the system B demonstrates that the ψ-function cannot be interpreted as a (complete)description of a physical state (of a single quantum mechanical object)"[19]

Einstein changed his position on the statistical character on Quantum Mechanics due to critique. Of course he was aware of the statistical character of Quantum Mechanics before[20] but the Born interpretation was an argument for incompleteness that was much easier to defend than the EPR argument.

The statistical character of Quantum Mechanics is connected to the notion of completeness. We will see in 4.3 that there are several definitions of completeness and there is a deep connection to the statistical character of Quantum Mechanics. Before discussing the connection between EPR and ER paper and based to the presented aspects before we will analyse cornerstones of Einstein's realism.

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[19]Translation by the author

[20]In his letter to Schrödinger where he explained the EPR argument he also refers to the statistical character[von Meyenn, 2010, letter 206,p.537]:

> "Nun beschreibe ich einen Zustand so: Die Wahrscheinlichkeit dafr, da die Kugel in der ersten Schachtel ist, ist $\frac{1}{2}$ Ist dies eine vollständige Beschreibung? Nein: Eine vollständige Aussage ist: die Kugel ist in der ersten Schachtel (oder ist nicht). So mu also die Charakterisierung des Zustandes bei vollständiger Beschreibung aussehen."

> "I describe a state this way: The probability for a ball being in the first box is $\frac{1}{2}$. Is that a complete description? No: A complete description is: The ball is in the first box (or is not). The representation of a complete state has to be that way." Translation by the author.

Cornerstones of Einstein's realism

The term reality cannot be seen isolated. It is connected to several conceptions that accompany reality, neither can a conception of reality of a scientist be seen as unvarying, it is subject of change and variation and often inconsistent[21]. A thorough discussion of Einstein realism would go beyond the scope of this paper, therefore we will only present aspects or cornerstones of the ideas of reality that Einstein had in mind. For a more detailed analysis of the evolution of Einstein's philosophical position over the years refer to [Howard, 1993], an in-depth discussion on Einstein's view on realism and philosophy of science is given by [Janssen and Lehner, 2014].

- Objectivity - There is a world outside, independent of our experiences, descriptions and ideas, a given reality that awaits to be uncovered. Connected to that Einstein also has in mind that there is a description that reflects this truth about nature. In that sense this view is contradictory in its claim to represent elements of the theory as human thoughts and ideals and simultaneously follow a correct path, as Einstein calls it. See section 2 "Einstein's view on theories and theory formation".

- (Mathematical) - Simplicity: Einstein has in mind that a physical theory has to fulfil simplicity [Einstein, 1934, p.165]:

  > *"It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements[22] as simple and as few as possible without having to surrender the adequate representation of a single datum of experience."*

  Moreover he assumed this simplicity is something that is realised in nature as well [Einstein, 1934, p.165]:

  > *"Our experience up to date justifies us in feeling sure that in Nature is actualized the ideal of mathematical simplicity."*

  This is a contradiction in the understanding of simplicity, because the ideas and elements of the theory should fulfil simplicity, but they are merely a construction by human minds, so this simplicity is nothing fundamental and cannot be the same simplicity he mentions in the second quotation. We presented a discussion of the concept of simplicity in context with interpretations of Quantum Mechanics and its peculiarities in [Krizek, 2017b]. For an account on Einstein's conception of simplicity refer to [Howard, 1993, p.230].

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[21]By this inconsistency we mean the contradiction between several ontological statements or concepts that refer to reality.

[22]By elements he refers to the Machian elements of the theory.

- Mathematical entities - Einstein favours the four dimensional continuum over algebraic structures with arbitrary or even infinite dimensionality. Out of some statements one could conclude that he took the position of naive realism in a sense of taking mathematical entities as elements of reality, a mathematical realism.

- Methodological and phenomenal realism - According to [Lehner, 2014] Einstein advanced a view on the role of invariants in theoretical models, he calls that position methodological realism. The role of invariants in a theory is a distinguished one, the invariants represent elements of the theory that are assumed to have a strong connection to objective reality. This conception is based on the theory of relativity but also found application in modern theoretical physics like in Gauge theories. In contrast to methodological realism he sees phenomenal realism [Lehner, 2014, p.313]:

    "By contrast, physical measurements do not reveal those objective entities themselves, but their representations in local coordinate frames, that is, noninvariant quantities. Such measurements are no less real, but they are real in a different sense: their reality is phenomenal, dependent on an observer situated in a specific way."

  Following Lehners reading it was Einstein's attempt to apply methodological realism to Quantum Mechanics, and the failure of this attempt was reason of reject Quantum Mechanics as a foundational theory.

- Deterministic character - Einstein was very clear about that point. To him a theory with statistical character is only provisional and cannot represent real events. Refer also to the discussion in section 3.4.

- Non-localizability: In some sense this non-localizability is a nonlocality feature, but not in the sense of an action-at-a-distance, more in the sense of the notion of a non-absolute localization of particles. The concept of non-localization was familiar to Einstein from the way the gravitational energy behaves in general theory of relativity. We will give a more detailed discussion of Non-localizability in section 4.2.

- Singularity-free - To Einstein the representation of particles in a field theory implied the problem of singularities of the field. To him a reasonable representation of particles has to be free from such singularities in the field. Refer also to the discussion in section 4.2.

- Separability: The physical reality allows to separate the world into parts that can be handled independently. To Einstein this is a prerequisite for scientific reasoning per se (See Don Howard [Howard, 1993, p.234ff] and [Howard, 2005, p.39]). The spatiotemporal separability consists of locality and separability of systems per se. There is also a separability without a spatiotemporal aspect, which is realised for instance by linear systems

that can be decomposed into systems that evolve independent of each other, even if they share a position n spacetime. Fourier analysis is a good example of this superposition principle that allows to decompose systems into separable subsystems.

- Locality and causality - In Einstein's view reality obeys locality in the sense of local realism: Events in two spacelike separated areas cannot affect each other. Therefore all meaningful descriptions of nature have to obey local realism as well.

- Unambiguity or uniqueness: A complete description[23] leaves no room for ambiguity. This is connected to aristotelian ideas of physics, basically the contradiction principle, which is based on common sense experience. This view reflects also in the EPR argument, in the condition of completeness [Einstein et al., 1935] and in later variations of the EPR argument [von Meyenn, 2010, p.537]; [Popper, 2005, p.482]; [Einstein, 1936, p.341]; [Einstein, 1948].

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[23]See section 4.3 for an overview on several definitions of completeness. This completeness will be defined as *theory completeness* and will refer to the elements of the theory.

4 EPR=ER in 1935 and its revival

In this chapter we provide an answer to the question if there is a connection between the EPR and ER paper and if yes in what kind of way. In a modern context, where in some interpretations of Quantum Mechanics nonlocality plays a crucial role, and the imprint by popular culture, it is a tempting thought to put the nonlocality features of Quantum Mechanics in connection to nonlocal structures like wormholes. It is even more surprising to find out that the first wormhole paper[24] was written by the same persons that challenged the completeness of Quantum Mechanics with the EPR argument, namely by Einstein and Rosen. The conclusion one could draw at first glance is close-at-hand: Einstein and Rosen could have had in mind to solve the issue with nonlocalities in Quantum Mechanics by wormholes. This conclusion is wrong due to several reasons we present now.

4.1 Nonlocality

At first one has to say that there was no issue with nonlocality to solve. At least not for Einstein. Einstein recognised the nonlocality of Quantum Mechanics which basically occurred in context with the projection postulate, but to him it was untenable to accept nonlocalities in physics, so he was not searching for a way to explain nonlocalities by an underlying theory. It is this following statement that makes clear that non-locality for the authors of the EPR-paper, and it can be assumed specially for Einstein, was not considered to be a possible or even thinkable part of ontology [Einstein et al., 1935]:

> "This makes the reality of P and Q depend upon the process of measurement carried out on the first system, which does, not disturb the second system in any way. No reasonable definition of reality could be expected to permit this."

The action-at-a-distance effects in Newtonian mechanics were a problem already to Newton and entail a lot of philosophical problems like difficulties with causality. It is comprehensible that Einstein was not considering to give up the beautiful new and self-consistent formulation of a causal structure. To him there must have been a different solution that would integrate the quantum effects in the framework of field theory with its complete[25] character. But field theory had its problems as well and Einstein was full aware of them.
If nonlocality was not the pursued goal of the ER-paper what was it then? In his paper on the method of theoretical physics he clarifies one of the main difficulties in field theory [Einstein, 1934]:

> "The most difficult point for such a field-theory at present is how to include the atomic structure of matter and energy. For the theory

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[24]Before the ER paper, only speculations by Flamm and Weyl can be found. As pointed out by [Visser, 1996] which coined the term "first wormhole paper".

[25]In the sense of *theory completeness*

*in its basic principles is not an atomic one in so far as it operates
exclusively with continuous functions of space, in contrast to classical
mechanics whose most important feature, the material point, squares
with the atomistic structure of matter."*

The absolute localization of particles was an open question to Einstein and we
will see the connection to the ER-paper in section 4.2.

4.2   Localizability of particles

In section 2 we presented what Einstein understood by the term "atomic struc-
ture": The singularity-free representation of particles in space. To him it was
not only a problem of field theory but had a deep connection to quantum theory
[Einstein, 1934]:

*"... it seems to me certain that we have to give up the notion of
an absolute localization of the particles in a theoretical model. This
seems to me to be the correct theoretical interpretation of Heisen-
berg's indeterminacy relation."*

Not only that the particle is represented singularity-free, the refutation of an
absolute localization of the particle was the goal of this concept[26]. To Einstein
it promised the reconstruction of quantum effects by general theory of relativity.
The ER-paper opens in the abstract with reference to this atomistic theory of
matter [Einstein and Rosen, 1935b]:

*"The writers investigate the possibility of an atomistic theory of mat-
ter and electricity which, while excluding singularities of the field,
makes use of no other variables than the $g_{\mu\nu}$ of the general relativity
theory and the $\phi_{\mu\nu}$ of the Maxwell theory."*

In the model of the Einstein-Rosen bridge the bridge is a representation of a
particle, but the position of the particle is smeared out. The position is in con-
trast to the external Schwarzschild-solution not given by the position coordinate
of the singularity. The metric around the particle is defined by the Einstein-
Rosen-bridge solution, still there is no account of the particles exact position.
This is satisfying in the sense of Einstein's requirement for solving the quantum
problem within a continuum theory presented in section 2 [Einstein, 1934].

It is comprehensible that the goal of the ER-paper was to present a field theory
that solves the localisation problem for particles. Even more is was pursued
to reconstruct quantum theory with a unified field theory that can claim to be
complete. The definition of completeness and its peculiarities will be presented
in section 4.3.
For a more detailed discussion concerning the philosophical implications of the
concept of particles refer to [Achinstein, 1991] and [Kuhlmann, 2010].

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[26]It is noticeable that this highly non-classical idea has been proposed by Einstein who
repeatedly had to face critique on his inflexible position and his adherence to classical physics.

4.3 Completeness

The completeness used in the EPR argument, ER paper and accompanying sources is a peculiar concept. At first hand one has a clear idea what is understood by completeness, but we will see that a clear definition is needed for the discussion of the context on the EPR argument. We will show that there are several definitions of completeness involved. The connection to the ER-paper will be presented.

4.3.1 Theory completeness

In 1934 Einstein defined the meaning of the term completeness [Einstein, 1934]:

> "A complete system of theoretical physics consists of concepts and basic laws to interrelate those concepts and of consequences to be derived by logical deduction."

In section 3.1 a notion of the structure of a physical theory has been presented. By using physical quantities, laws to connect them, and concepts to give structure and principles to the theory, a complete theory in this sense can be pictured as following:
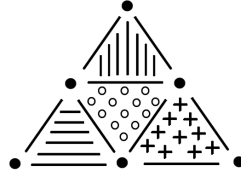

Figure 7: Structure of a theory

The mentioned elements (physical quantities, laws and concepts/principles) are meant in the Machian sense and per se not connected to an ontology. Einstein emphasized that these elements are "...free inventions of the human mind..." [Einstein, 1934] and therefore pure constructions of thought. Therefore this definition of completeness can be understood more as a consistency in the theoretical system. In any sense it forgoes to give ontological statements. We will call this completeness *theory completeness*.

An application of this theory completeness is given in [Einstein and Rosen, 1935b]:

> "One of the imperfections of the original relativistic theory of gravitation was that as a field theory it was not complete; it introduced the independent postulate that the law of motion of a particle is given by the equation of the geodesic. A complete field theory knows only

> *fields and not the concepts of particle and motion. For these must not exist independently of the field but are to be treated as part of it."*

We want to give a definition for theory completeness that gets as close as possible to the concept Einstein had in mind:

Definition 1. *A theory fulfils theory completeness if its physical quantities and laws are connected by concepts that only relate to each other and the theory renounces extra assumptions of concepts, that exceed the framework of the theory.*

Theory completeness does not refer to an ontology. It is only referring to elements of the theory and requests a consistency in the used concepts. Einstein refers to it in [Einstein and Rosen, 1935b] as Closeness(Vollständigkeit).
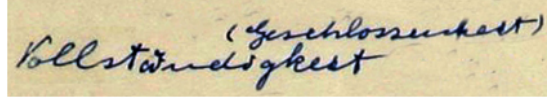
Figure 8: Completeness - Closeness [Einstein and Rosen, 1935a]

References to the *theory completeness* can be found in: [Einstein, 1934, p.164,166]; [Einstein et al., 1935, p.778]; [Einstein and Rosen, 1935b, p.76, p.77]; [Einstein, 1936, p.316, p.344]; [Einstein, 1948, p.320, p.323].

4.3.2 Bijective completeness

The EPR paper posed a question on completeness, but is it *theory completeness* that is challenged? If we refer to the EPR paper a definition of completeness is given [Einstein et al., 1935]:

> *"Whatever the meaning assigned to the term complete, the following requirement for a complete theory seems to be a necessary one: every element of the physical reality must have a counterpart in the physical theory"*

As can be seen immediately this definition of completeness differs from the definition of *theory completeness*, it even more extends the term completeness to the ontological level, whereas the *theory completeness* definition refers only to the theory and its elements. The latter relates the elements of reality to the elements of a theory. [Fine, 2009] calls this completeness *bijective completeness*. Following [Einstein et al., 1935] a definition would be:

Definition 2. *Bijective completeness means that every element of the physical reality ϵ_i must have a counterpart t_i in the physical theory.*

Though *bijective completeness* is a Bijection between elements of the theory and elements of reality it starts with the latter ones. Starting at elements of reality
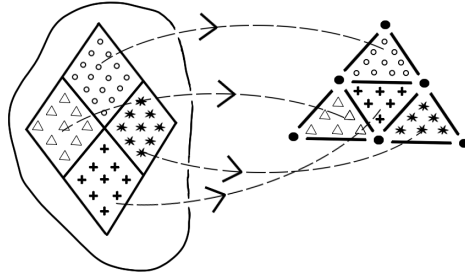
Figure 9: Bijective completeness

causes the problem how to identify them and the corresponding elements of the theory. For this purpose [Einstein et al., 1935] had to assume a criterion of reality to identify these elements of reality[27].

From an accompanying source one more aspect about *bijective completeness* can be seen. Karl Popper received a letter from Einstein in September of 1935, where Einstein explained the idea of the EPR paper in a short and very clear way. One more aspect of Einstein's notion of *bijective completeness* can be found there [Popper, 2005, p.413]:

> *"Since a complete description of a physical state must necessarily be an unambiguous description (apart from superficialities such as units, choice of the co-ordinates etc.), it is therefore not possible to regard the ψ-function as the complete description of the state of the system."*

He concludes that a *bijective completeness* description has to be unambiguous, so if every element of the physical reality has to have a(one) correspondence in the theory is must be vice versa. This unambiguousness assumption is not presented explicit like the reality criterion, the completeness criterion, or separability/locality, though it plays an important role in the EPR argument.
It is further interesting that Einstein seem to have seen a problem with the pure *bijective completeness* argument in the EPR paper. Superficialities as choice of coordinates as we have presented them before in section 3.4 would not fulfill *bijective completeness*, so Einstein clarifies this to Popper [Popper, 2005, p.413].

Einstein gives no exact definition of completeness in the EPR paper, but states that definition 2 of *bijective completeness* is a necessary requirement for his concept of completeness. Is it possible that *theory completeness* is the completeness
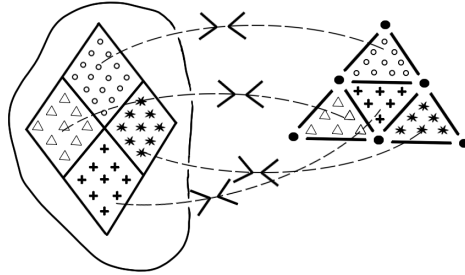
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[27]See also Section 3.4.

Figure 10: Bijective completeness and unambiguousness

Einstein had in mind? *Theory completeness* is only defined by elements of the theory, and makes no statement about the ontology of the theory, therefore it can be ruled out that *bijective completeness* is a necessary requirement of *theory completeness*. *Bijective completeness* would have to be true, so that *theory completeness* could be true, but it is obvious that the structure of the elements of a theory is completely independent of any mapping ϕ between the elements of the theory and elements of reality. *Theory completeness* is not the completeness Einstein had in mind for whom he used *bijective completeness* as a necessary requirement.

References to the *bijective completeness* can be found in [Einstein et al., 1935, p.777, p.778, p.780] and [Einstein, 1936, p.341].

4.3.3   Born completeness

Since *theory completeness* is excluded to be the completeness meant in the EPR paper there must be a different definition of completeness. [Fine, 2009] identifies two further concepts of completeness: *Born completeness* and *Schrödinger completeness*:

> *"According to the Bornian conception, a complete description is essentially nonprobabilistic; genuinely probabilistic assertions are necessarily incomplete."*

These conceptions of completeness can be identified in letters from Einstein to Schrödinger where Einstein also stated the "Einstein's boxes" Gedankenexperiment, which discusses the probabilities of finding a ball in a box [von Meyenn, 2010, letter 206, p.537].

Definition 3. *Born completeness for a theory is fulfilled if the theory contains no probabilistic elements. A theory which contains statistical accounts and probabilistic statements is considered to be incomplete.*

Following this definition Quantum mechanics is incomplete per definition. No further arguments are needed.

Is *Born completeness* the completeness Einstein meant in the EPR paper? If *Born completeness* would be addressed it would mean that the authors of the EPR paper had in mind that without *bijective completeness Born completeness* could not be true. That follows out of the EPR condition of completeness.
On the other hand the statement "Quantum mechanics fulfils *Born completeness*" is false. In this case out of the rules for the material conditional it follows that independent of the truth value of the *bijective completeness* the statement "*theory completeness* is a necessary requirement for *Born completeness*" is true. To show the truth value of *bijective completeness* is therefore meaningless and we can rule out that *Born completeness* is the meant conception of completeness in the EPR paper.

References to the *Born completeness* can be found in [Einstein, 1936, p.339, p.341, p.343, p.347].

4.3.4   Schrödinger completeness

[Fine, 2009, p.71] gives an analysis of *Schrödinger completeness*:

> "*By contrast the Schrödinger view is that probabilities can be fundamental, not to be reduced to something else. Thus the Schrödinger conception is that a complete description of a state of affairs can be a probabilistic assertion, with probability less than unity, which (somehow) tells the whole truth about that state of affairs. If there were some further truth to be told, then the probabilistic assertion would be an incomplete description*"

Its getting a bit vague with *Schrödinger completeness*, because it seems that Einstein's conception of it is not a very clear one, at least he did not define it explicitly. What can be seen immediately is that the definitions of probability in context with *Born completeness* and *Schrödinger completeness* differ vastly. Whereas the concept of the probability of occurrence of outcomes in an ensemble of events is used in the first conception, the latter defines probability as fundamental and related to single events. The interpretation of probability will go beyond the scope of this work, we will therefore stick to an operational definition of *Schrödinger completeness* that is based on Einstein's considerations in his letter to Schrödinger [von Meyenn, 2010, letter 206, p.537]:

Definition 4. *Schrödinger completeness for a theory is fulfilled if the theory contains probabilistic statements about elements of the theory. The probabilistic statements refer not to the statistical ensemble of many events, but to one single event. The probabilistic statements are fundamental in the sense, that nothing more can be said about the system. The theory is considered to be incomplete if there are potential new elements[28] in the theory that would abandon the probabilistic character of the theory.*

The definition of *Schrödinger completeness* refers to what we would nowadays call "hidden variables".
It is difficult to decide if *Schrödinger completeness* is the completeness Einstein is referring to in the EPR paper. If we carefully analyse the argument in the letter to Schrödinger from the 19th of June 1935 we find a self reference in the definition of completeness [von Meyenn, 2010, letter206, p.537]:

> *"Nun beschreibe ich einen Zustand so: Die Wahrscheinlichkeit dafür, da die Kugel in der ersten Schachtel ist, ist $\frac{1}{2}$ Ist dies eine vollständige Beschreibung? Nein: ... Ja: ... Der Zustand vor dem Aufklappen ist durch die Zahl $\frac{1}{2}$ vollständig charakterisiert, deren Sinn sich bei Vornahme von Beobachtungen allerdings nur als statistischer Befund manifestiert."*

> *"I describe a state this way: The probability for a ball being in the first box is $\frac{1}{2}$. Is that a complete description? No: ... Yes: ... The state before opening the box is completely characterised by the number $\frac{1}{2}$, whose meaning becomes manifest as statistical account by applying measurements on the system."*

What is meant by this complete characterisation by the number $\frac{1}{2}$? If we assume it to be *bijective completeness* in a way that the number is ascribed to an element of reality, and the number is interpreted as probability in an epistemic way, this would represent an epistemic account of the situation. Depending on the interpretation of probability it would represent propensity, knowledge about the state of the ball, or degrees of belief on its state.

In principle this would be a description that would fulfill *bijective completeness* because one can give a Bijection to elements of reality in each of these accounts. Therefore it is plausible to identify *Schrödinger completeness* as the completeness Einstein challenged in the EPR paper.

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[28]Einstein calls them "fremde Faktoren"(external factors). The translation of "fremd" is peculiar, since in the literally translation it would be "strange", "weird", "alien" or "different", but in context it refers to factors that are not considered in the theory. They are not part of the theory, so we decided to use external. In our modern context these factors could be understood as "hidden variables"

4.3.5 ψ - completeness

In an account to classify hidden variable models [Harrigan and Spekkens, 2010] proposed a definition of ψ-completeness:

> "An ontological model is ψ-complete if the ontic state space Λ is isomorphic to the projective Hilbert space $\mathcal{PH}$ (the space of rays of Hilbert space) and if every preparation procedure P_ψ associated in quantum theory with a given ray ψ is associated in the ontological model with a Dirac delta function centered at the ontic state Λ that is isomorphic to Ψ, $p(\lambda|P_\psi) = \delta(\lambda - \lambda_\psi)$."

This definition of ψ-completeness is claimed to be identical with *bijective completeness* [Harrigan and Spekkens, 2010, p.147]:

> "It is quite clear that by "real state of the real system", Einstein is referring to the ontic state pertaining to a system. Bearing this in mind, his definition of completeness can be identified as precisely our notion of ψ-completeness given in Definition 2"

This is a misunderstanding of Einstein's view on completeness and elements of reality. Einstein is not using the wording elements of reality in his letter to Schrödinger[von Meyenn, 2010, letter206, p.538], but referencing to *bijective completeness*:

> "Man beschreibt in der Quantentheorie einen wirklichen Zustand eines Systems durch eine normierte Funktion ψ der Koordinaten (des Konfigurationsraumes).Die zeitliche Änderung ist durch die Schrödinger-Gleichung eindeutig gegeben. Man möchte nun gerne folgendes sagen: ψ ist dem wirklichen Zustand des wirklichen Systems eindeutig zugeordnet. Der statistische Charakter der Meßergebnisse fällt ausschlielich auf das Konto der Messapparate bzw. des Prozesses der Messung. Wenn dies geht rede ich von einer vollständigen Beschreibung der Wirklichkeit durch die Theorie. Wenn aber eine solche Interpretation nicht durchführbar ist, nenne ich die theoretische Beschreibung unvollständig."

> "In quantum theory a real state of a system is described by a normed function ψ of coordinates (of configuration space). The evolution in time is given by Schrödinger equation uniquely. One would like to say: ψ is corresponding to the real state of the real system uniquely. The statistical character of the measurement results is a consequence of the measurement apparatus respectively the measurement process. If this works, I speak about a complete description of reality by a theory. If such an interpretation is not possible, I call the theoretical description incomplete.[29] "

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[29]Translation by the author

[Harrigan and Spekkens, 2010] conclude out of these statements that Einstein's *bijective completeness* is identical to their *psi-completeness*, but the latter one is setting up an Isomorphism between the projective Hilbert space and an ontic state space, whereas *bijective completeness* is a Bijection between elements of a theory and elements of reality. These elements of reality have no per se defined mathematical structure. In our proposed classification scheme for interpretations of Quantum Mechanics the elements of reality are located in level four, the ontological level. Assuming that the elements of reality are elements of an ontic state space is a kind of mathematical realism, but we think that is not the intention of the claim.

On the other hand [Harrigan and Spekkens, 2010] have good reasons to assume that Einstein by referring to the "real state of the real system" [von Meyenn, 2010, letter 206, p. 538] means elements of a theory that would replace Quantum Mechanics. They propose an argument for Einstein's preference for the EPR argument in contrast to his incompleteness argument from 1927. One aim according to them was Einstein's goal to emphasize the epistemic character of the state vector and to look out for hidden variable theories.

ψ-*completeness* is a useful classification tool for hidden variable theories, but we refuse the identity of ψ-*completeness* and *bijective completeness* based on the involvement of completely different structures in these two conceptions. This does not affect any applications of ψ-*completeness* in [Harrigan and Spekkens, 2010].

4.3.6 Standard completeness

The following completeness criterion is not part of the original EPR discussion and is presented here to amend the overview on accounts on completeness. [Held, 2012] gives an argument against *standard completeness*. They bring the timeframe of the involved events in consideration and claim a conflict between *standard completeness* and the principle that "QM should deliver probabilities for physical systems possessing properties at definite times" [Held, 2015].

4.3.7 Remarks on completeness and the EPR-ER connection

The targets of the EPR paper are manifold, one aim is to show that *Schrödinger completeness* does not hold for the unitary core of Quantum Mechanics itself. Einstein claimed that Quantum Mechanics did not receive its statistical character out of the measurement process. The unitary evolution of the state vector itself is according to him incomplete in the sense of *Schrödinger completeness*. The critique on Quantum Mechanics is one story, but more interesting in our context is the motivation for showing this incompleteness. Replacing a successful theory with a new unknown theory without a good reason encounters opposition. Einstein had to convince the community of the incompleteness of Quantum Mechanics to present a new program [Einstein et al., 1935]:

> "While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible."

Einstein retreated quickly, already in 1936, to the position of *Born completeness* which to him was sufficient to show the need for a theory that fulfils his requirements. Referring to his article [Einstein, 1948] he notes in a letter to Besso [Speziali et al., 1972, p.403]:

> "Es freut mich dass Du meinen kleinen Aufsatz gelesen hast. Hast Du auch gemerkt wie unlogisch Pauli darauf geantwortet hat? Er leugnet es, dass diese Art der Beschreibung unvollständig sei, sagt aber im selben Atemzuge, dass die ψ Funktion eine statistische Beschreibung des Systems sei, die Beschreibung einer System-Gesamtheit. Dies ist doch nur eine andere Form der Aussage. Die Beschreibung des (individuellen) Einzelsystems ist unvollständig! "

> " I am pleased to hear that you read my little article. Have you recognised also how illogical Pauli responded? He denies that this description is incomplete, but states in the same breath that the ψ function is a statistical description of the system, the description of a system-totality. This is just a different formulation of the statement: The description of an (individual) single-system is incomplete![30] "

This statement is the essence of the *Born completeness*. And it shows that the EPR arguments and all its successor arguments by Einstein had one main purpose: To show the incompleteness of Quantum Mechanics for the specific purpose to justify the need for a new field theory to replace Quantum Mechanics. Despite the ambiguous notions of completeness it is fair to conclude that it was the idea to present this new complete theory in the next published work following 2 month later after the EPR paper; the Einstein-Rosen bridge paper.

We presented the meaning of the terms "completeness" and "reality" and pointed out which aspects of reality were specially relevant to Einstein in context with the 1935 papers. In the following chapter we will make use of these clarifications to discuss the connection of the 1935 papers in the context of the unified field theory program.

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[30]Translation by the author

4.4 The Unified field theory program

Einstein had in mind to resolve the incomplete[31] statistical character of quantum theory with a unified field theory. Even in 1934 one of the conclusive statements of his paper on the method of theoretical physics addresses the statistical character [Einstein, 1934]:

> "I still believe in the possibility of giving a model of reality, a theory, that is to say, which shall represent events themselves and not merely the probability of their occurrence."

We have seen before that to Einstein the concept of completeness is strongly connected to the statistical character of quantum theory. In the original EPR argument in [Einstein et al., 1935] he used the reality criterion, locality(Trennungsprinzip - separation principle), and the contradiction principle to conclude the incompleteness of Quantum Mechanics. In [Einstein, 1936] he did not refer to those principles any more, instead he argued with the ensemble interpretation, which is an indirect use of the *Born completeness*. According to the ensemble interpretation Quantum mechanics only refers to a statistical ensemble to events, not to a single event itself. To Einstein this argument was sufficient to show the incomplete character of Quantum Mechanics and to head for an underlying theory. We can see that this is one of the main agendas of the EPR paper [Einstein et al., 1935]:

> "While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible."

This new theory he presented in the ER-paper, at first independent of the EPR critique, was meant to solve the problem of the atomistic character of matter and fulfils Einstein's requirement of completeness [Einstein and Rosen, 1935b]:

> "In favor of the theory[32] one can say that it explains the <u>atomistic character</u> of matter as well as the circumstance that there exist no negative neutral masses, that it introduces no new variables other than the $g_{\mu\nu}$ and $\phi_{\mu\nu}$, and that in principle it can claim to be <u>complete</u> (or closed). On the other hand one does not see a priori whether the theory contains the quantum phenomena." [our underlining]

Here he means *theory completeness* which is evident when he puts the term "closed" in brackets to underline that it refers to the closeness of a unified field theory that hopefully solves the particle problem.

It is remarkable that Einstein differs between the atomistic character of the theory and the question if the theory contains quantum phenomena. We have seen

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[31]The completeness refers to *Born completeness*
[32]The Einstein-Rosen bridge model for elementary particles

before what he means exactly by "atomistic character", it refers to the problem of the localization of particles without accepting singularities of the field. The Einstein-Rosen bridge would have solved this in a way satisfactorily to Einstein. It is not exactly clear what Einstein understands by the term "quantum phenomena" and how the theory could contain them. It is comprehensible that he means the quantum states and the formalism of Quantum Mechanics, which cannot be found in the Einstein-Rosen bridge representation for a particle, but it certainly was Einstein's hope that the quantum effects would emerge out of this new model.

In [Einstein, 1936] he presented the EPR argument in a modified form [33]in line with the ER-bridge. Here the connection of these two papers as part of a program is evident:

> "Es wird aber gezeigt, dass die Überzeugung von der Unfähigkeit der Feldtheorie, diese Probleme mit ihren Methoden zu 1ösen, auf Vorurteilen beruht."

> "It will be shown that the conviction of the inability of field theory to solve these problems[34] rests on a prejudice."

and

> "Angesichts dieser Sachlage erscheint es mir durchaus gerechtfertigt, die Frage ernsthaft zu erwägen, ob nicht doch die Grundlage der Feldphysik mit den Quanten-Tatsachen vereinbar ist."

> "In view of this situation it seems appropriate to me to reconsider the question if a unification of the foundations of field theory and quantum facts is possible[35]."

This sums up the scientific program Einstein attempted to present. At the beginning he refers to a comparison between Quantum Mechanics and statistical mechanics and states that a statistical or effective theory is not an appropriate starting point to develop a complete[36] theory. In his view Quantum Mechanics was this effective theory that delivers only statistical predictions, but gives no account for the single events.

In a more private environment, a letter from the 16th February 1936 to his friend Michele Besso, Einstein expresses this point of view explicitly. Since to my knowledge this letter has not been cited in context with the EPR discussion we want to cite it in full length[37] [Speziali et al., 1972, p.308].

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[33]See Appendix ??.

[34]The problems refer to the inability of field theory to give account for a molecular structure and to describe quantum phenomena with methods of field theory [Einstein, 1936, p.347]

[35]Translation by the author

[36]In the sense of *Born completeness* and *theory completeness*

[37]We let aside a passage on the political situation in Europe

"Ich halte die statistische Physik trotz all ihrer Erfolge doch für eine vorübergehende Phase und habe Hoffnung, zu einer wirklich befriedigenden Theorie der Materie zu gelangen. Ich sende Dir gleichzeitig eine kurze Arbeit, die den ersten Schritt darstellt. Das neutrale und das elektrische Teilchen erscheinen gewissermassen als Loch im Raume, derart, dass das metrische Feld in sich selbst zurückkehrt. Der Raum wird als zweischalig dargestellt. In der Schwarzschildschen strengen zentralsymmetrischen Lösung erscheint das Teilchen im gewöhnlichen Raume als Singularität vom Typus $1 - \frac{2m}{r}$. Durch die Substitution $1 - 2m = u^2$ wird das Feld regulär im r-Raume. Wandert u von $-\infty$ bis $+\infty$, so wandert r von $+\infty$ zu $r = 2m$ und hierauf wieder zurück zu $r = +\infty$. So kommen beide "Blätter" im Riemannschen Sinne zustande, die an der "Brücke" $r = 2m$ bezw. $u = 0$ stetig zusammenhängen. Aehnlich bei der Elektrizität. Die Aufgabe an der ich mit einem jungen Kollegen (russischer Jude) unablässig schwitze ist die Behandlung des Mehrkörperproblems auf dieser Basis. Wir haben aber die ernsthaften Schwierigkeiten des Problems bereits überwunden, sodass sich bald zeigen wird was daran ist. Jedenfalls ist es eine wundervolle mathematische Aufgabe."

" To me statistical physics despite its success is a transitory phase and I have hope that we arrive at a really satisfying theory of matter. I send you enclosed a short work, that represents a first step. The neutral and the electrical particle appear as a hole in space, of this kind that the metric field returns to itself. Space is represented as two sheets. In the strict spherical symmetric solution of Schwarzschild the particle appears in usual space as singularity of the kind $1 - \frac{2m}{r}$. By substitution $1 - 2m = u^2$ the field becomes regular in r-space. If u goes from $-\infty$ to $+\infty$, r is going from $+\infty$ to $r = 2m$ and back again to $r = +\infty$. This represents both "sheets", that are connected by the "bridge" at $r = 2m$ respectively $u = 0$. Likewise as it is in electricity. The challenge a young college (russian jew) and I labour away over is the many-body-problem on that basis. We have conquered the serious problems already, so it will show soon if there is something serious about it. Anyway it is a wonderful mathematical problem.[38] "

The connection between Einstein's critique on Quantum Mechanics, specifically its statistical character as well as the problems of the localizability of particles, and the connection to the Einstein-Rosen bridge approach could not be closer than presented here.

Einsteins unified field theory program failed for reasons we will not discuss in this work. For a comprehensible presentation of this field theoretic program refer to [Van Dongen, 2010]. Einstein expressed his hopes how the program

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[38]Translation by the author

would succeed and how it might contain the quantum phenomena [Einstein, 1936]:

> *"Erst die Untersuchung des Mehr-Brücken-Problems kann zeigen, ob diese theoretische Methode eine Erklärung für die empirisch erwiesene Massengleichheit der Teilchen in der Natur liefert, und ob sie den von der Quantenmechanik so wunderbar erfassten Tatsachen gerecht wird."*

> *"Only the examination of the many-bridge-problem can show if this theoretical model provides an explanation for the empirical proven equality of masses of particles in nature, and if it can reproduce the facts that are represented by Quantum Mechanics in such a delightful way[39]."*

These hopes of Einstein seem to be carried on in recent program claims that reconsider the idea of a deep connection between entanglement and Einstein-Rosen bridge solutions.

4.5 Recent claims

Recents claims bring back the EPR-ER idea in the context of Quantum Gravity [Maldacena and Susskind, 2013]:

> *"General relativity contains solutions in which two distant black holes are connected through the interior via a wormhole, or Einstein-Rosen bridge. These solutions can be interpreted as maximally entangled states of two black holes that form a complex EPR pair. We suggest that similar bridges might be present for more general entangled states."*

and [Susskind, 2016]:

> *"Now I feel that our current views of Quantum Mechanics are provisional; it's the best we can do without a much deeper understanding of its connection with gravity, but it's not final. The reason involves a very particular development, the so called ER=EPR principle. ER=EPR tells us that the immensely complicated network of entangled subsystems that comprises the universe is also an immensely complicated (and technically complex) network of Einstein-Rosen bridges."*

It is remarkable that the critique by Susskind sounds not that different from Einstein's reservations on Quantum Mechanics. The term "provisional" and the statement that Quantum Mechanics is not to be seen as final can be understood as synonymous to Einstein's claims for incompleteness of Quantum Mechanics in the sense of *theory completeness.*

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[39]Translation by the author

5 Conclusion

It is Einstein's merit that he raised questions about the foundations of Quantum Mechanics which are still relevant and bother contemporary physics. In this light it is more than just history of science to reconsider the old arguments. The EPR-argument is such an argument that is still relevant to the discussions on the interpretation of Quantum Mechanics. If one browses through Einstein's papers of the year 1935, the ER-paper coincides with the EPR-paper and a closer look reveals that there are further interrelations.

We presented several notions of completeness that can be identified in the EPR-paper, the ER-paper and its accompanying sources: *Theory completeness* which only refers to the elements of the theory, *bijective completeness*, or the condition of completeness as it was named in the EPR-paper, *Born completeness* and *Schrödinger completeness*. Recent conceptions of ψ-*completeness* and *standard completeness* amend the presentation. We argued that ψ-*completeness* is not identical to *bijective completeness* based on the type of elements the both notions of completeness refer to.

Einstein's intention of writing the EPR paper was to demonstrate the incompleteness of Quantum Mechanics[40] to justify his ambitions to look for a complete (in the sense of *theory completeness*) theory. This new theory should fulfil *theory completeness* and *Born completeness*, explain the atomistic character of Quantum Mechanics, which turned out to be the singularity-free description of particles, and reproduce the quantum phenomena, which refers to the structure and formalism of Quantum Mechanics.

With the ER-paper Einstein presented this new theory, which developed into his attempts to present a unified field theory. His critique on Quantum Mechanics lasted parallel to his ongoing work on the unified field theory, but focused later on a position that was easier to hold. He criticized merely *Born completeness*, because to him it was undeniable that a new theory is needed and he shifted the effort from critique on Quantum Mechanics to development of a unified field theory.

The tempting thought to explain nonlocal phenomena inside Quantum Mechanics with ER-bridges is nothing Einstein would have considered, because to him separability was a principle applied to show that Quantum Mechanics is incomplete in the sense of *Schrödinger completeness* independently of a measurement carried out on the system.
By applying the separation principle and exclusion of the measurement process from the consideration, Einstein aimed to show that the statistical character of Quantum Mechanics is not owed to the measurement process, but resides only

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[40]We identified *Schrödinger completeness* as the completeness he challenged in the EPR paper

in the state vector formalism and the Schrödinger evolution. The state vector formalism is fully deterministic and by that he concludes on the incompleteness of the formalism of Quantum Mechanics.

In the literature about the EPR-argument the statistical character of Quantum Mechanics plays a key-role in Einstein's critique on Quantum Mechanics, and it truly deserves this role. We emphasized one further essential motivation for Einstein's attempt to reconstruct Quantum Mechanics, the "atomistic structure", or as what we identified it: the problem of an absolute localization of particles. In classical field theory the attempts to give a localization of particles result in field singularities. For Einstein this problem was an incompleteness in general relativity in the sense of *theory completeness*. He aimed to solve this problem with a new complete theory, which he presented in first outlines in the ER-paper. The ER-bridge was merely intended to solve the problem of localization of particles, the ER-brigde would also provide a singularity-free representation of particles and according to Einstein would be consistent with Heisenberg's uncertainty principle. It must have been insights like this that encouraged him to hope that this approach is promising that quantum phenomena might emerge in his new theory and by that reconstruct Quantum Mechanics out of field theory.

Unfortunately these hopes of Einstein failed and until the end of his life he could not present a satisfying solution on this problem. Though we presented that Einstein never had in mind to resolve nonlocality issues in Quantum Mechanics by Einstein-Rosen bridges, because of his not-acceptance of nonlocality as a feasible concept in a theory.

Recent programmatic approaches that claim a EPR=ER principle have been presented. We cannot estimate if the EPR=ER approaches will contribute to the development of a unified theory of quantum gravity, but the idea is promising and if the program would turn out to be successful it would be a late gratification for Einstein's field theory program to unify Quantum Mechanics and gravity, even if this success was not in the originally intended direction. It would also underline the extent of Einstein's intuition, that brought forward foundational problems in the interpretation of Quantum Mechanics and probably indicated the right direction in this quest.

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